Can Fixed Regular Deposits Overcome Savings Constraints?*

Anett John†

Version April 2016

Abstract

Empirical evidence from developing countries suggests that there is a high demand for informal savings mechanisms even though these often feature negative returns - such as deposit collectors, ROSCAs, microloans, and informal borrowing. Why do people not just save at home, instead of relying on such costly devices? In a savings model with hyperbolic discounting and uncertainty, I show why a commitment to fixed regular savings deposits can help individuals to achieve the welfare-maximising level of savings, when they would not be able to do so on their own. Such regular-instalment commitment products further increase welfare by smoothing savings contributions. The setting is enriched by endogenising take-up, and giving individuals the ability to choose their own commitment stakes. The results point to the possibility that the observed demand for costly informal savings devices may simply represent a demand for commitment savings products with fixed periodic contributions, as they are commonly offered by banks in rich countries.

Keywords: commitment savings, hyperbolic discounting, regular instalments
JEL classification: C93, D03, D14, O12

*This article is a shortened version of chapter 2 of my dissertation. A longer version was previously circulated under the name Anett Hofmann, and the title "Just A Few Cents Each Day: Can Fixed Regular Deposits Overcome Savings Constraints? - Evidence from a Commitment Savings Product in Bangladesh." I would like to thank my advisors, Oriana Bandiera, Maitreesh Ghatak, and Gharad Bryan, for their invaluable support and advice. I am grateful for comments from Dean Karlan, Stefano DellaVigna, Matthew Levy, George Loewenstein, and Johannes Spinnewijn. Thanks to Jonathan de Quidt and Erina Ytsma, as well as to various seminar audiences, for helpful comments and discussions. All errors and omissions are my own.
†CREST, email: anett.john@ensae.fr.

1 Introduction

There are many reasons why even individuals with very low incomes have the need to convert small everyday sums of money into larger lump sums. Among them are life-cycle needs (such as marriage, childbirth, education or home building), insurance against emergencies (like sickness, loss of employment, natural disasters) and business opportunities (e.g. buying land or a new machine). To obtain these lump sums, individuals use a variety of financial services – many of which are very costly. Observational evidence from developing countries indicates a popularity of borrowing for consumption purposes, as well as for periodic business expenses (e.g., to maintain stock).[1] In both cases, obtaining a loan is not essential to generating the income that will repay it: Many clients go through a loan cycle every month for years, paying back in small instalments every week. A savings cycle with frequent contributions differs from this in one initial loan disbursement, which pales in size compared to annualized interest rates between 100 and 500 percent.[2] Given such interest rates, the decision to borrow rather than to save seems puzzling.

A similar puzzle is the popularity of costly or inflexible savings mechanisms. The United States has a long tradition of Christmas Clubs - savings programs which ask clients to deposit small amounts of money each week until December, at low or zero interest rates, and fees for early withdrawal. First recorded in 1909, they continued to attract 10 million clients in 1995 (Crosgrave (1927), Laibson et al. (1998)). In a developing context, the wandering deposit collectors found in South Asia and Africa charge *negative* interest rates in return for collecting people's savings.[3] Similarly, rotating savings and credit organizations (ROSCAs) are prevalent all across the developing world (e.g. Besley et al. (1993), Gugerty (2007)). While ROSCAs are costless, they are inflexible to an individual member's needs.

Why do people not just save at home, instead of relying on such costly devices? An answer suggested in the existing literature are (quasi-)hyperbolic preferences.[4] Hyperbolic discounters are impatient over current trade-offs (now vs. tomorrow) and patient over future trade-offs (one year vs. one year plus one day). As a result, they procrastinate saving. This leads either to failure to reach their savings target, or to a failure to smooth consumption by saving too much in the last minute. If hyperbolic individuals realize their time inconsistency, they will be willing to pay for commitment products which force them to save. Recent empirical literature has confirmed a link between hyperbolic discounting and a preference for commitment (see Ashraf et al. (2006), Bauer et al. (2012), and Gugerty (2007)).

This paper shows theoretically how hyperbolic discounters can benefit from a commitment to fixed regular savings deposits. In doing so, it suggests that the empirical popularity of inflexible ROSCAs, costly deposit collectors, Christmas Clubs, and (partly) of microcredit and informal borrowing may simply represent a demand for commitment savings products with fixed periodic contributions (hereafter: "regular saver product"). Using a model of hyperbolic discounting, it is shown that individuals cannot reach their welfare-maximising level of savings at home, and that a regular saver product can increase their achievable savings level, as well as smooth savings contributions over time.

The existing literature on formal commitment savings has focused largely on savings products featuring withdrawal restrictions, but without an obligation to deposit in the first place.[5] For a hyperbolic

[1] See e.g., Collins et al. (2009), the Indian vegetable vendors studied in Ananth et al. (2007), or Rutherford (2000).
[2] See e.g., Rutherford (2000) or Ananth et al. (2007).
[3] See e.g. Besley (1995) and Rutherford (2000).
[4] Most prominently, Laibson (1997) and O'Donoghue and Rabin (1999).
[5] See e.g. Ashraf et al. (2006), Brune et al. (2011), Karlan et al. (2010), and Giné et al. (2010).

discounter, simply giving him the opportunity to restrict his withdrawals will not do - he will be prevented from touching past savings, but will have no incentive to contribute further. A demand for commitment to frequent instalments has been previously hypothesised in Rutherford (2000) and Bauer et al. (2012), but not modelled theoretically. A related theoretical contribution is Fischer and Ghatak (2010), who show in a deterministic setting that incentive-compatible loan sizes increase with repayment frequency. Related empirical contributions include Dupas and Robinson (2013) and Kast et al. (2014), who study social commitments to regular deposits, as well as Benartzi and Thaler (2004), who study 401(k) pension contributions with default options. In a very recent contribution, Afzal et al. (2015) test the savings-versus-loan choice in a lab-in-the-field setting.

In a savings framework that includes uncertainty as well as a motive for consumption smoothing, I show the effects of a regular-instalment product on savings levels, consumption paths, and welfare. The adoption decision is endogenised. Given fully sophisticated agents, offering a regular saver product increases the range of preferences where a savings goal (a nondivisible good) can be reached. Savings contributions are smoothed towards a balanced savings path. The setting is enriched by allowing individuals to choose their own "commitment stakes", specifically, the penalty they pay upon defaulting on the savings plan. An extension shows the effects on the demand for loans.

Uncertainty plays a key role in the model: In a deterministic setting, commitment is costless. There are no disadvantages to imposing infinitely large penalties on undesirable behaviour. Thus, as long as such commitments are technically feasible, the agent can always achieve the first-best savings behaviour by imposing large penalties on non-compliance. More realistically, individuals face uncertain environments, which create a demand for flexibility. For instance, the arrival of an income shock may mean that saving is no longer optimal. Amador et al. (2006) study this trade-off between commitment and flexibility in a generalised setting, where the planner directly controls the choice set. In the current setting, where commitment is implemented using penalties, it creates an interesting dynamic interaction: Conditional on being unable to save for a nondivisible expense in autarky, the most time-inconsistent individuals are the least likely to adopt commitment, since they require the largest stakes.

The model builds on Basu (2014), who focuses on withdrawal-restriction products and supply-side variations. It is consistent with Amador et al. (2006)'s general result that trade-offs between commitment and flexibility are optimally solved with minimum-savings requirements. Finally, it generalizes the model in John (2015) for multiple periods, concave utility, and regular instalments.

2 Theory: Hyperbolic Discounting and Regular Instalments

The following section attempts to provide a theoretical foundation which justifies the need to provide regular saver commitment products when agents have time-inconsistent preferences and face uncertainty. The benchmark model without banking services is based on Basu (2014), but departs from it in the design of the commitment savings product as well as in the presence of shocks.

2.1 The Model

Consider an agent who decides whether to save up for a nondivisible good (such as school tuition fees, a new roof, or a medical treatment) which costs $2 < p < 3$ and yields a monetary benefit $b > 3$. The agent lives for three periods and receives a per period income of $y_t = 1$, which he can either consume immediately or save.[6] He cannot borrow. His instantaneous utility function is concave and satisfies the Inada conditions, specifically, $u'(c) > 0$, $u''(c) < 0$, and $u'(0) = \infty$. His lifetime utility as evaluated in each period is the discounted sum of the instantaneous utilities:[7]

$$U_t = u(c_t) + \beta \sum_{k=t+1}^{3} u(c_k)$$

When $\beta < 1$, the discount rate between the current period t and $t+1$ is lower than the discount rate between subsequent periods – i.e. the agent exhibits a *present bias*. The model will assume that he is *sophisticated* – i.e., he realizes that his future selves will display the same present bias that he has, resulting in an inconsistency between the preferences of his current self and those of his future selves. The model further assumes that there is no other form of discounting ($\delta = 1$), and that the gross return on savings is $\rho = 1$ in both the autarky and the banking scenario. This isolates the effect of a commitment to regular instalments, since it is not profitable for individuals to use banking services simply to gain higher returns.

The agent faces uncertainty through income shocks: Each period, his income is lost with a probability of λ. A single shock is sufficient to reduce lifetime income to 2, thus rendering the nondivisible good unattainable. The presence of shocks in the model thus captures a demand for flexibility. Interpretations of the shock range from genuine income shocks (unemployment, bad business) to unexpected vital expenses (hospital bills). More generally, λ captures the probability that saving for the nondivisible good ceases to be optimal for any time-consistent reason. Once hit by a shock, any existing savings are optimally spread over the remaining periods for consumption.

While the literature disagrees on how to evaluate the welfare of hyperbolic discounters, the paper follows O'Donoghue and Rabin (1999): The welfare of a hyperbolic discounter is what he would like to maximise in a hypothetical period 0, i.e., just before the start of his life: $W = U_0 = u(c_1) + u(c_2) + u(c_3)$. I assume throughout that b, p are such that it is welfare-maximising to save for the nondivisible.[8]

2.2 Autarky

Denote by s_t the cumulative savings sent from period t to $t+1$. The following analysis assumes that no shock has hit up to period t. If a shock does hit (i.e., if $y_t = 0$), the agent immediately gives up any plans to save for the nondivisible, and instead spreads available savings s_{t-1} optimally over the remaining periods. The first-best benchmark can be inferred from the behaviour of a time-consistent agent: An agent with $\beta = 1$ will always buy the nondivisible, absent shocks. For $\lambda = 0$, the optimal savings schedule is easy to infer: Given the agent's desire to smooth consumption over time, it is optimal for him to split the necessary savings burden of $p - 1$ evenly over periods 1 and 2, and then spend his period 3 income plus accumulated savings on the good. The implied savings profile is $s_1 = \frac{p-1}{2} \equiv \bar{s}$, $s_2 = p - 1 = 2\bar{s}$. For $\lambda > 0$, there is a precautionary savings motive,[9] even if the agent does not intend

[6] Three periods are sufficient to illustrate the main mechanism of the model, which deals largely with the savings coordination between periods 1 and 2. It is straightforward to extend the model to $N \geq 3$ periods, in which case the savings burden is allocated between periods 1 and $N-1$ (in period N, the choice is trivial). All main results hold with N periods.

[7] The initial model setup is partially quoted from John (2015). While the present paper uses a three-period model and concave utility to study the effect of regular instalments, John (2015) uses a two-period model and linear utility to show the effect of partial sophistication on commitment choices more generally.

[8] More specifically, I assume that b, p are such that desirability holds given fixed equal savings instalments, $\bar{s} \equiv \frac{p-1}{2}$.

[9] Since expected income is $1 - \lambda$, and realised income absent shocks is 1, this precautionary savings motive follows from $u''(c) < 0$, and does not require assumptions on $u'''(c)$.

to save for the nondivisible. Denote such precautionary savings s_t^{No}. It can be shown that the optimal savings path is slightly increasing, i.e., $s_1 < \bar{s}$.[10]

If $\beta < 1$, the three period selves engage in strategic interaction. Savings behaviour can be analysed using backward induction.

Period 3 The decision in period 3 is mechanical: The agent buys the good whenever his assets are sufficient, i.e., whenever $s_2 \geq p - 1$, and absent shocks. Excess savings, as well as savings not sufficient to buy the good, are consumed.

Period 2 The period 2 self knows that his future self will buy the good if and only if he saves $s_2 \geq p - 1$, and absent shocks. He decides whether to send $s_2 = p - 1$, in which case the good is bought, or less. Given $\beta \leq 1$, it is never optimal to send $s_2 > p - 1 > 1$. If the agent prefers not to save for the good, he will want to smooth s_1 over periods 2 and 3 – denote this $s_2^{No}(s_1) = argmax(u(y_2 + s_1 - s_2) + \beta E[u(y_3 + s_2)]$ subject to $0 \leq s_2 < p - 1$. This equation also describes his savings behaviour in case of a shock, where $y_2 = 0$. It is easy to show that s_2^{No} increases in β.

Lemma 1. *(a) The period 2 agent is willing to save for the nondivisible and transfer $s_2 = p - 1$ if s_1 is bigger than some threshold value, $s_1 \geq s_{min}$. (b) s_{min} is strictly decreasing in the time-inconsistency parameter β. (c) The effect of the shock frequency λ on s_{min} is ambiguous.*

(All proofs are in Appendix B.)

Period 1 Analogue to the minimum s_1 threshold for period 2, it is useful to identify the maximum s_1 that period 1 is willing to save, conditional on purchase of the nondivisible. Denote this as s_{max}. If this maximum is bigger than the minimum required (and no shocks arrive), the good will be purchased. Conditional on the nondivisible *not* being purchased (i.e., period 2 saves $s_2^{No} < p - 1$), period 1 saves only for precautionary purposes: $s_1^{No} = argmax(u(y_1 - s_1) + \beta E[u(y_2 + s_1 - s_2^{No}) + u(y_3 + s_2^{No})])$ for $s_1^{No} \geq 0$ and $y_t = \{0, 1\}$. The occurrence of a shock implies $y_1 = 0$ and thus $s_1^{No} = 0$.

Lemma 2. *The maximum that period 1 would be willing to save, denoted s_{max}, is strictly increasing in the time-inconsistency parameter β.*

The interiority of s_{max} in the relevant range of β follows from the desirability of the nondivisible for a time-consistent agent: We know that $s_{max}(\beta = 1) \geq \frac{p-1}{2}$ and that $s_{max}(0) = 0$. The final component to the autarky equilibrium is the *optimal* way in which period 1 would like to allocate the savings burden of $p - 1$ across periods 1 and 2:

Lemma 3. *The optimal allocation of savings from period 1's perspective, denoted $s_1 = s_{opt}$, is strictly increasing in β, and always smaller than s_{max}.*

It is interesting to note that s_{opt} contains the term $\partial s_2^{No} / \partial s_1 > 0$ as a result of the agent's time-inconsistency (with time-consistent preferences, the envelope condition would apply): s_2^{No} is chosen optimally given period 2's preferences, which makes it suboptimal from period 1's perspective for $\beta < 1$. As a result, s_1 has a first-order positive effect on s_2^{No} (see Appendix B for details).

[10] The probability of remaining shock-free (and thus obtaining the nondivisible) increases over time, from $(1 - \lambda)^3$ ex-ante to $(1 - \lambda)$ once period 2 has been reached without a shock. This makes it optimal to slightly skew the savings burden $p - 1$ towards period 2. To see this formally, note that expected utility decreases in s_1 when evaluated at $s_1 = \bar{s}$: $dU/ds_1 = (1 - \lambda)^2[-u'(1 - s_1) + u'(2 + s_1 - p)] + (1 - \lambda)\lambda[-u'(1 - s_1) + u'(s_1 - s_2^{No})] < 0$ for $s_1 = \frac{p-1}{2} > 0.5$. By the envelope condition, $dU/ds_1 = \frac{\partial U}{\partial s_1} + \frac{\partial U}{\partial s_2} \cdot \frac{\partial s_2}{\partial s_1} = \frac{\partial U}{\partial s_1}$.

Autarky Equilibrium

Given a decreasing $s_{min}(\beta)$ and an increasing $s_{max}(\beta)$-function, there is a threshold level $\hat{\beta}$ such that $s_{min}(\beta) \leq s_{max}(\beta)$ for any $\beta \geq \hat{\beta}$. The fact that $\hat{\beta}$ is in the relevant interval (0, 1] follows from $s_{min}(0) > s_{max}(0)$ and $s_{min}(1) \leq s_{max}(1)$: The former follows from $s_{min}(0) > 1$, $s_{max}(0) = 0$. The latter is a consequence of desirability, by which a time-consistent agent always purchases the good. Since the different period selves are perfectly able to anticipate each other's behaviour, the nondivisible will be purchased (absent shocks) for all $\beta \in [\hat{\beta}, 1]$. Absent shocks, equilibrium savings are

$$s_1 = \begin{cases} max(s_{min}, s_{opt}) & if\ \beta \in [\hat{\beta}, 1] \\ s_1^{No} & if\ \beta \in [0, \hat{\beta}) \end{cases}, \quad s_2 = \begin{cases} p-1 & if\ \beta \in [\hat{\beta}, 1] \\ s_2^{No} & if\ \beta \in [0, \hat{\beta}). \end{cases}$$

The equilibrium savings path is illustrated in Figure 1. If a shock occurs in any period, the individual gives up any plans to save for the nondivisible, and smoothes available assets $y_t + s_{t-1}$ over future periods, saving $s_t^{No} \geq 0$ for all t after the shock. Importantly for the later analysis with commitment, it is ambiguous whether autarky savings will be above or below $\bar{s} \equiv \frac{p-1}{2}$. Considering that a time-consistent agent saves $\bar{s}$ (for $\lambda = 0$) or slightly below $\bar{s}$ (for $\lambda > 0$), this question corresponds to O'Donoghue and Rabin's (1999) *pre-emptive overcontrol*: A sophisticated hyperbolic discounter may both save more or less than a time-consistent agent, depending on the numerical values used for $(b - p)$ and $u''(c)$. In the following, autarky scenarios with $s_{min}(\hat{\beta}) = s_{max}(\hat{\beta}) > \bar{s}$ will be characterized as "overcontrol scenarios," whereas those with $s_{min}(\hat{\beta}) = s_{max}(\hat{\beta}) < \bar{s}$ will be referred to as “procrastination scenarios.” Either term refers to the equilibrium savings at the threshold level $\hat{\beta}$, regardless of the savings made at other levels of β.

2.3 Equilibrium with a Regular Saver Commitment Product

The autarky equilibrium is inefficient for $\beta < \hat{\beta}$: The agent is unable to save for the nondivisible good, despite its desirability. The following section investigates the effect of offering agents a commitment to fixed regular savings contributions - as commonly found in loan contracts, pension savings, and other forms of regular saving.[11]

The Regular Saver product is defined as follows: Consider an agent who can commit in period 0 to deposit a fixed amount $\bar{s} = \frac{p-1}{2}$ in a bank account in both period 1 and 2. He also chooses a default penalty D, subject only to a limited liability constraint which prevents negative consumption. Once the agent fails to deposit $\bar{s}$ in a period, he is charged the default penalty D, but immediately receives back any accumulated savings. In addition, he is free to save at home independently of his bank contributions. His total cumulated savings (in the bank plus at home) can be captured as s_t. The penalty D is imposed in period 1 if $s_1 < \bar{s}$, and in period 2 if $s_1 \geq \bar{s}$, $s_2 < 2\bar{s}$. The contract is successfully completed with $s_1 \geq \bar{s}$, $s_2 \geq 2\bar{s}$. The assumption that the contract is signed in period 0 simplifies things greatly, as the agent is not subject to temptation in this period, but it requires the bank to have contract-enforcing power. As before, the savings outcome can be derived using backwards induction. The commitment adoption decision in period 0 is discussed at the end of this section.

[11] Footnote 10 argues that the first-best savings schedule is slightly increasing for $\lambda > 0$, i.e., $s_1 < \bar{s}$. For small λ, this effect is likely to be small. Commitment products with increasing savings schedules are possible, but may pose serious challenges to institutional implementation: The first-best schedule will depend on individual values of λ, $u''(c)$, p and b. The present analysis focuses on fixed-instalment products due to their empirical popularity and ease of administration.

Period 3 Period 3 behaviour is identical to that in autarky. The agent will buy the nondivisible whenever he can afford it, i.e., whenever $s_2 \geq p - 1$ holds, and absent shocks.

Period 2 Suppose period 1 has not been hit by a shock, and has transferred $s_1 \geq \bar{s}$. Suppose further that a shock hits in period 2: At an asset level of $s_1 < 1$ and contractual savings of $s_2 = 2\bar{s} = p - 1$, default is unavoidable. The resulting consumption level is $c_2 = s_1 - D - s_2^{No} \geq 0$, implying that a penalty of $D \leq s_1$ can be enforced. Absent shocks, period 2 is faced with the decision of whether to send $s_2 = 2\bar{s} = p - 1$. He is willing to do so if he receives an s_1 that satisfies

$$u(1 + s_1 - (p-1)) + \beta[(1-\lambda)u(b) + \lambda u(p-1)] \geq u(1 + s_1 - D - s_2^{No}) + \beta E[u(y_3 + s_2^{No})] \quad (1)$$

Since the inequality differs from the autarky case only in the penalty D, the same proof can be used to show that the nondivisible is bought for any $s_1 \geq s_{min}^B$. The threshold $s_{min}^B(\beta)$ will be strictly lower than $s_{min}(\beta)$ in the autarky case: The right-hand side of the inequality decreases when D is introduced, while the left-hand side stays unchanged. The effect of the penalty disappears for $s_1 < \bar{s}$: Period 1 has already defaulted on the contract and paid the penalty, so the contract is no longer active in period 2. As a result, $s_{min}^B(\beta) = s_{min}(\beta)$ for $s_1 < \bar{s}$.[12] Finally, in the region where $s_{min}^B(\beta) > \bar{s}$, the period 2 agent is not willing to save for the nondivisible unless period 1 makes additional savings at home.

Period 1 Consider the maximum s_1 that period 1 is willing to save, once subjected to a penalty for $s_1 < \bar{s}$. Limited liability implies that the penalty cannot be enforced if there is a shock: With no income or previous savings, $c_1 = s_1 = 0$. Absent shocks, period 1 prefers to save for the nondivisible if

$$\begin{aligned} u(1 - s_1) &+ \beta(1-\lambda)^2(u(2 + s_1 - p) + u(b)) \\ &+ \beta(1-\lambda)\lambda(u(2 + s_1 - p) + u(p-1)) \\ &+ \beta\lambda(u(s_1 - D - s_2^{No}) + E[u(y_3 + s_2^{No})]) \\ &\geq u(1 - D - s_1^{No}) + \beta E[u(y_2 + s_1^{No} - s_2^{No}) + u(y_3 + s_2^{No})]. \end{aligned} \quad (2)$$

In contrast to the inequality for s_{min}^B, both sides of the s_{max}^B-inequality are affected by the penalty. Even for a devoted saver, the penalty is unavoidable if a shock hits in period 2, causing the left-hand side to decrease in D (discounted by $\beta\lambda$). On the right-hand side, the penalty is the consequence of a deliberate decision to default in period 1.

Lemma 4. *For small shock frequencies λ, and in the region where savings are skewed towards period 1, $s_1 \geq \bar{s} \equiv \frac{p-1}{2}$, adopting a regular-instalment product increases the maximum the agent is willing to save, i.e., $s_{max}^B > s_{max}$. A sufficient constraint on the shock frequency is $\lambda < \frac{u'(1)}{u'(0.5)}$. In the region $s_1 < \bar{s}$, adopting the regular-instalment product unambiguously decreases s_{max}.*

Note that inequality 2 is specific to the region $s_1 \geq \bar{s}$: The penalty is not charged in period 1 if the agent saves for the nondivisible. Consider the case where necessary savings are $s_1 < \bar{s}$, i.e., period 1 could ensure the good is bought even if he does not contribute $\bar{s}$. In this case, he faces a penalty

[12] The two sections of the $s_{min}^B(\beta)$-function combine with a horizontal line at $s_{min}^B(\beta) = \bar{s} = \frac{p-1}{2}$. In this region, the s_{min} required by period 2 is lower than $\bar{s}$ if he faces the penalty, and higher than $\bar{s}$ if he does not. To keep the contract active and ensure that period 2 faces the penalty, the period 1 agent needs to save $s_1 \geq \bar{s}$.

whether or not he saves for the good. The penalty D enters in period 1 on both sides of the inequality. The resulting threshold $s^B_{max}(\beta)$ is strictly *lower* than the original threshold $s_{max}(\beta)$.

As before, the final component is the *optimal* way in which period 1 would like to split the savings burden, denoted $s^B_{opt}(\beta)$, when subjected to the penalty. In autarky, time-inconsistency and a positive shock frequency $\lambda > 0$ imply that s_{opt} is strictly below $\bar{s}$. In order to maintain this preference for pushing the lion's share of the savings burden to period 2, period 1 now needs to pay a "premium" of D. With his consumption reduced to $c_1 = 1 - s^B_{opt} - D$, period 1 wants to save even less than in autarky, implying $s^B_{opt} < s_{opt}$.

Alternatively, period 1 may prefer to jump to $s_1 = \bar{s}$ to avoid the penalty. Doing so is mechanically dominant in the region of s_1 where $s_1 + D > \bar{s}$. For lower levels of savings, the agent weighs the current cost of $\bar{s}$ (instead of $s_1 + D$) against the benefit of consuming D more in the next period. This results in a dominated region $[\hat{s}(\beta), \bar{s})$ for some threshold $\hat{s}(\beta) < \bar{s} - D$, illustrated in Figure 2. Note that this dominated region is only defined for β such that $s^B_{max}(\beta) \geq \bar{s}$, because it requires that the agent prefers to save $\bar{s}$ over giving up on the nondivisible good altogether.

Lemma 5. *(a) The optimal allocation of savings from period 1's perspective decreases with the adoption of a regular-instalment product, i.e.* $s^B_{opt} < s_{opt}$. *(b) However, there exists a threshold* $\hat{s}(\beta)$ *such that saving in the region* $\hat{s}(\beta) < s_1 < \bar{s}$ *is strictly dominated by saving in balanced instalments,* $s_1 = \bar{s}$. *(c) As* β *increases, a larger range of savings* $s_1 \in (\hat{s}(\beta), \bar{s})$ *is strictly dominated. Equivalently,* $\hat{s}(\beta)$ *weakly decreases in* β.

Equilibrium and Contract Choice

The nondivisible is purchased whenever $s^B_{max}(\beta) \geq s^B_{min}(\beta)$, which occurs for any $\beta \in [\hat{\beta}_B, 1]$. Equilibrium savings (absent shocks) are analogue to those for autarky, except for a lower savings threshold $\hat{\beta}_B < \hat{\beta}$, and a dominated region $s_1 \in [\hat{s}(\beta), \bar{s})$:

$$s_1 = \begin{cases} max(s^B_{min}, s^B_{opt}) & if\ \beta \in [\hat{\beta}_B, 1] \text{ and } max(s^B_{min}, s^B_{opt}) \notin [\hat{s}, \bar{s}) \\ \bar{s} & if\ \beta \in [\hat{\beta}_B, 1] \text{ and } max(s^B_{min}, s^B_{opt}) \in [\hat{s}, \bar{s}) \\ s^{No}_1 & if\ \beta \in [0, \hat{\beta}_B) \end{cases}, \quad s_2 = \begin{cases} p - 1 & if\ \beta \in [\hat{\beta}_B, 1] \\ s^{No}_2 & if\ \beta \in [0, \hat{\beta}_B). \end{cases}$$

Figure 2 shows the equilibrium savings path in autarky settings characterized by overcontrol, while Figure 3 starts from autarky settings with procrastination (as defined in Section 2.2).

Proposition 1. *Offering a regular saver commitment product makes the nondivisible good achievable under a larger range of preferences,* $\beta \in [\hat{\beta}_B, 1]$, *instead of* $\beta \in [\hat{\beta}, 1]$ *in the autarky benchmark. A lower threshold* $\hat{\beta}_B \leq \hat{\beta}$ *emerges for any positive penalty* $D > 0$ *if the autarky setting is characterized by pre-emptive overcontrol. In settings characterized by procrastination, a sufficiently large penalty* D *is required to guarantee* $\hat{\beta}_B \leq \hat{\beta}$.

Proposition 2. *For those already able to save in autarky,* $\beta \geq \hat{\beta}$, *the adoption of a regular saver product weakly smoothes savings contributions (and thus consumption) towards* $\bar{s}$.

Period 0 Adoption Decision and Penalty Choice In principle, any agent with $\beta \in [0, 1)$ can benefit from commitment. Given a sufficiently large penalty, it makes the nondivisible achievable and smoothes savings: Absent shocks, the contract is trivially enforceable in period 1 if $D > \bar{s}$, and in period 2 if $D > 2\bar{s}$. As a result, the threshold $\hat{\beta}_B$ can be moved to an arbitrarily low β. The downside of commitment is the risk of "rational default" due to shock frequency λ: The penalty not only acts to discipline the agent when income is available, it also needs to be paid when the agent no longer finds it welfare-maximising (or feasible) to save for the nondivisible. Limited liability implies that this risk is limited to shocks in period 2: If a shock hits in period 1, the agent has no assets or income, thus the penalty cannot be enforced. In period 3, the contract is no longer active.

The resulting decision is a two-step problem: The period 0 agent first decides which penalty D offers the optimal trade-off between commitment and flexibility (denoted D^*). He then makes a binary choice between adopting the regular saver product with the optimal penalty, or not adopting the product. The choice of D^* is complex: Due to the consumption smoothing motive, D^* is non-monotonic in β, and sensitive to the autarky scenario. To see this, define $D_{min}(\beta)$ to be the minimum effective penalty which achieves $s^B_{max}(\beta) \geq s^B_{min}(\beta)$, and thus makes the nondivisible good achievable. In overcontrol settings, D_{min} generally results in equilibrium savings that are skewed towards period 1 (as at $\beta = \hat{\beta}_B$ in Figure 2). Increasing D further smoothes savings towards $\bar{s}$. The associated benefit is small and continuous, as opposed to the large discrete benefit from obtaining the nondivisible good. Whether the agent deems it worthwhile to increase D beyond D_{min} will depend on the involved uncertainty λ as well as his preference for consumption smoothing $u''(c)$. Finally, non-monotonicity of the optimal penalty D^* follows from the non-monotonicity of the equilibrium savings path in autarky, $s_1 = max(s_{min}, s_{opt})$. Individuals who are able to save in autarky (those with $\beta \geq \hat{\beta}$) can use D to create a dominated region $[\hat{s}(\beta), \bar{s})$ that includes their current savings level s_1, causing them to jump to $\bar{s}$. As β increases, autarky equilibrium savings first decrease along s_{min}, and then increase along s_{opt}. Analogously, the penalty required to make the agent jump to $\bar{s}$ first increases and then decreases with β.[13] As a result, the commitment adoption decision is likely to be non-monotonic for $\beta \in [\hat{\beta}, 1)$.

For the sake of simplicity, I abstract from the consumption smoothing motive, and focus on the range of $\beta \in [0, \hat{\beta})$. For this range, the nondivisible is not achievable in autarky, and obtaining it constitutes the primary benefit of the Regular Saver product. This focus is empirically meaningful: It restricts the analysis to the part of the population who are not able to save for lump-sum consumption expenditures by themselves. Given full sophistication, a Regular Saver contract with a penalty D_{min} will enable the agent to save for the nondivisible (absent shocks). By construction, $D_{min} = 0$ for $\beta \geq \hat{\beta}$.

Proposition 3. *For a given shock frequency λ, the minimum effective penalty D_{min} that will enforce saving weakly decreases in the time-inconsistency parameter β.*

Proposition 4. *Optimal Penalty Choice: (a) In autarky settings characterized by procrastination, the minimum penalty required to obtain the nondivisible good, D_{min}, also guarantees perfect consumption smoothing, with $s_1 = \bar{s} \equiv \frac{p-1}{2}$ and $s_2 = 2\bar{s} = p - 1$. Consequently, the optimal contract is to choose D_{min}. (b) In autarky settings characterized by overcontrol, D_{min} fails to guarantee consumption smoothing. The optimal contract involves $D \geq D_{min}$, with equilibrium savings weakly skewed towards period 1 ($s_1 \geq \bar{s}$).*

In settings involving pre-emptive overcontrol, penalties exceeding D_{min} are most likely to be chosen when the motive for consumption smoothing is large. This occurs for high autarky savings $s_{min}(\hat{\beta}) = s_{max}(\hat{\beta}) \gg \bar{s}$, small shock frequencies λ (so the agent is less averse to big penalties), and large nondivisible prices p (increasing the benefits to consumption smoothing).

Having determined D^*, the period 0 agent faces the binary decision of whether or not to take up the Regular Saver product. The adoption decision is characterized in Proposition 5.

[13] Formally, the agent will choose D such that $\hat{s}(\beta) = max(s^B_{min}, s^B_{opt})$ holds exactly.

Proposition 5. *Commitment Adoption Decision: (a) Adoption of the Regular Saver product decreases in the shock frequency λ, the nondivisible price p, and the required penalty D_{min}. Adoption increases in the nondivisible good's benefit b. (b) Conditional on being unable to save in autarky ($\beta < \hat{\beta}$), those with the highest amount of time-inconsistency are the least likely to adopt commitment (i.e., adoption increases in β), as the most time-inconsistent agents require the largest penalties.*

The key to Proposition 5 is that the benefit of an effective commitment contract (obtaining the nondivisible good at smooth savings $s_1 = \bar{s}$) is independent of β: The period 0 agent bases his adoption decision on the welfare function $W = E[u(c_1) + u(c_2) + u(c_3)]$, which does not directly depend on β.[14] Put simply, the time-inconsistency parameter β determines how difficult it is for the agent to save for the nondivisible, but not how much he benefits from achieving it. As a result, agents with low β (and thus a high required penalty D_{min}) will find commitment unattractive in expectation.

Welfare Implications Given full sophistication, agents assess the optimal degree of commitment D^* correctly. Everyone who adopts the commitment product is made better off in expectation. Ex-post welfare losses occur due to shocks, causing a fraction λ of adopters to default each period. In summary, offering regular-instalment savings products is weakly welfare-increasing for sophisticated hyperbolic discounters, and strictly welfare-increasing for those who adopt it.

2.4 Loans

The introductory section hypothesized that individuals may use loans to obtain a commitment to fixed regular instalments, when (cheaper) savings products with such commitment features are not available. This section formalises this argument. As before, suppose that the individual has a consumption opportunity in period 3, which costs p and yields benefit b. Extend the time horizon to five periods: The agent can either save for the good in periods 1 and 2, or he can take a loan in period 3, and repay it in periods 4 and 5. The timing of the consumption opportunity is the same in both cases in order to reflect the 'save before or borrow after' decision of many anticipated expenditures (such as school fees). To illustrate the simplest possible mechanism, abstract from uncertainty, and assume that an incentive-compatible loan enforcement mechanism is available.[15]

Assume a benchmark setting like that described in Section 2.2, except that loans are now available. The agent can borrow $p - 1$ at a per-period gross interest rate of R in order to buy the nondivisible good in period 3. Repayment follows in two equal instalments of $\frac{R^2}{R+1} \cdot (p-1)$ in periods 4 and 5.[16] An argument similar to that for the savings product can be used to show that frequent loan instalments are preferable to one single repayment from both enforcement and welfare perspectives (see also Fischer and Ghatak (2010)). It is convenient to define $L \equiv u(b) + u(1 - \frac{R^2}{R+1} \cdot (p-1)) + u(1 - \frac{R^2}{R+1} \cdot (p-1))$ for the utility derived from loans in periods 3 to 5. Finally, assume R and b such that the individual prefers loan-financed consumption over not consuming the good, i.e. $L \geq 3u(1)$.

[14] The result that the benefit obtained from commitment is independent of β holds exactly for $\lambda = 0$, and to a first-order approximation for $\lambda > 0$. For $\lambda > 0$, the agent values precautionary savings, which decrease in β. Commitment thus provides an insurance value which increases in β. However, this is a second-order effect which is quantitatively dominated by the offputting effect of a higher required penalty D_{min}.

[15] All basic conclusions will hold when allowing for uncertainty. However, the marginal trade-off between savings and loan products will shift with differences in the enforcement technology, since the good is consumed before the loan is repaid.

[16] The calculation of two equal instalments I follows from $I + R \cdot I = R^2(p-1)$.

The savings analysis is akin to that of the previous sections, except that credit-financed consumption replaces 'not consuming the good' as the outside option. As in the autarky setting (Section 2.2), the welfare-maximising choice is to save for the nondivisible good at home, which is cheaper than a loan. However, the period 1 and 2 selves disagree about how to split the savings burden between them. A new s_{min} and $s_{max}-$curve result, with a new threshold $\beta \geq \hat{\beta}_L$, above which the agent is able to save independently. Since the loan provides a weakly better outside option than not consuming the good, the range of β where the agent saves at home is reduced, i.e. $\hat{\beta}_L \geq \hat{\beta}$. Agents with $\beta < \hat{\beta}_L$ resort to loan financing. Notably, a monopolist moneylender could set R such that $L = 3u(1)$ holds with equality, implying $\hat{\beta}_L = \hat{\beta}$. In this case, there is no welfare benefit from the availability of loans.

Introducing a commitment savings product into this setting is analogous to Section 2.3: A fully sophisticated period 0 planner will sign a commitment contract with an optimal penalty D^*, which ensures that he can reach the nondivisible good without resorting to expensive loans, and which helps to smooth his savings contributions. In this simplified setting, the savings product is preferable to the loan for all $R > 1$, noting that the planner values all periods equally. Adding exponential discounting ($\delta < 1$), uncertainty, and differences in enforcement technology will affect the marginal trade-off between savings and loans. However, the basic result remains: In absence of commitment savings products, a time-inconsistent individual is likely to use loans to overcome savings constraints. As illustrated in Section 1, these loans can be very expensive, especially when used in repeated cycles. The introduction of a regular-instalment product provides an alternative, cheaper means to overcome savings constraints, and thus increases the agent's welfare.

3 Discussion

In attempting to explain the large observed demand for costly informal savings devices, this paper has outlined the benefits of regular-instalment commitment savings products for hyperbolic discounters. Specifically, such products can help to make lump-sum expenses achievable, encourage balanced savings paths, and reduce the demand for high-interest loans. Commitment devices with fixed regular instalments are relatively easy to implement in rich countries, where people's financial lives are largely automated. Any commercial bank will allow a client to set up a regular monthly direct debit from his income, before he has the opportunity to spend it. Such commitments are 'soft', in that they do not feature penalties for cancelling the service. Instead, they work via mental accounting, default option effects, and transaction costs (see Benartzi and Thaler (2004) for an application to pension contributions in the U.S.). Introducing formal commitments to regular instalments is more challenging in developing countries, where the use of direct debits is much less common. In many cases, people receive their income in cash, and need to walk to the bank to make regular deposits. In such cases, the use of self-chosen default penalties can be useful to implement commitment in practice. An empirical companion paper, John (2015), introduces and studies such a regular-instalment product in the Philippines, and finds large increases on savings levels on average. However, the study also serves as a cautionary note: In order to adopt welfare-increasing commitment products, individuals need to be sophisticated about their time-inconsistency. If, instead, sophistication is incomplete, then agents may choose commitment products which are too weak to make them save, leading to welfare losses. Thus, attention needs to be paid to the suitability of commitment products for the preferences of those who adopt them.

References

Afzal, Uzma, Giovanna d'Adda, Marcel Fafchamps, Simon Quinn, and Farah Said, "Two Sides of the Same Rupee? Comparing Demand for Microcredit and Microsaving in a Framed Field Experiment in Rural Pakistan," Working Paper, Centre for the Study of African Economies 2015.

Amador, Manuel, Iván Werning, and George-Marios Angeletos, "Commitment vs. Flexibility," *Econometrica*, 2006, *74(2)*, 365–396.

Ananth, Bindu, Dean Karlan, and Sendhil Mullainathan, "Microentrepreneurs and Their Money: Three Anomalies," Working Paper, Financial Access Initiative 2007.

Ashraf, Nava, Dean Karlan, and Wesley Yin, "Tying Odysseus to the Mast: Evidence from a Commitment Savings Product in the Philippines," *Quarterly Journal of Economics*, 2006, *121(2)*, 635–672.

Basu, Karna, "Commitment Savings in Informal Banking Markets," *Journal of Development Economics*, 2014, *107*, 97–111.

Bauer, Michal, Julie Chytilova, and Jonathan Morduch, "Behavioral Foundations of Microcredit: Experimental and Survey Evidence from Rural India," *American Economic Review*, 2012, *102(2)*, 1118–1139.

Benartzi, Shlomo and Richard H. Thaler, "Save More Tomorrow: Using Behavioral Economics to Increase Employee Saving," *Journal of Political Economy*, 2004, *112(1)*, 164–187.

Besley, Timothy, "Savings, Credit and Insurance," in Jere Behrman and T.N. Srinivasan, eds., *Handbook of Development Economics*, Vol. 3A, Elsevier, 1995.

_ , Stephen Coate, and Glenn Loury, "The Economics of Rotating Savings and Credit Associations," *American Economic Review*, 1993, *83(4)*, 792–810.

Brune, Lasse, Xavier Giné, Jessica Goldberg, and Dean Yang, "Commitments to Save: A Field Experiment in Rural Malawi," Policy Research Working Paper Series 5748, World Bank 2011.

Collins, Daryl, Jonathan Morduch, Stuart Rutherford, and Orlanda Ruthven, *Portfolios of the Poor - How the World's Poor Live on $2 a Day*, Princeton: Princeton University Press, 2009.

Crosgrave, Lloyd M, "Christmas Clubs," *The Quarterly Journal of Economics*, 1927, *41(4)*, 732–739.

Dupas, Pascaline and Jonathan Robinson, "Why Don't the Poor Save More? Evidence from Health Savings Experiments," *American Economic Review*, 2013, *103(4)*, 1138–71.

Fischer, Greg and Maitreesh Ghatak, "Repayment Frequency in Microfinance Contracts with Present-Biased Borrowers," EOPP Discussion Papers 21, London School of Economics 2010.

Giné, Xavier, Dean Karlan, and Jonathan Zinman, "Put Your Money Where Your Butt Is: A Commitment Contract for Smoking Cessation," *American Economic Journal: Applied Economics*, 2010, *2(4)*, 213–235.

Gugerty, Mary Kay, "You Can't Save Alone: Commitment in Rotating Savings and Credit Associations in Kenya," *Economic Development and Cultural Change*, 2007, *55(2)*, 251–282.

John, Anett, "When Commitment Fails - Evidence from a Regular Saver Product in the Philippines," EOPP Discussion Papers 55, London School of Economics 2015.

Karlan, Dean, Margaret McConnell, Sendhil Mullainathan, and Jonathan Zinman, "Getting to the Top of Mind: How Reminders Increase Saving," Working Paper 16205, National Bureau of Economic Research July 2010.

Kast, Felipe, Stephan Meier, and Dina Pomeranz, "Under-Savers Anonymous: Evidence on Self-Help Groups and Peer Pressure as a Savings Commitment Device," Working Paper 20239, National Bureau of Economic Research 2014.

Laibson, David, "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, 1997, *112(2)*, 443–478.

Laibson, David I, Andrea Repetto, Jeremy Tobacman, Robert E Hall, William G Gale, and George A Akerlof, "Self-control and saving for retirement," *Brookings papers on economic activity*, 1998, *1*, 91–196.

O'Donoghue, Ted and Matthew Rabin, "Doing It Now or Later," *American Economic Review*, 1999, *89(1)*, 103–124.

Rutherford, Stuart, *The Poor and Their Money*, Oxford: Oxford University Press, 2000.

A Figures

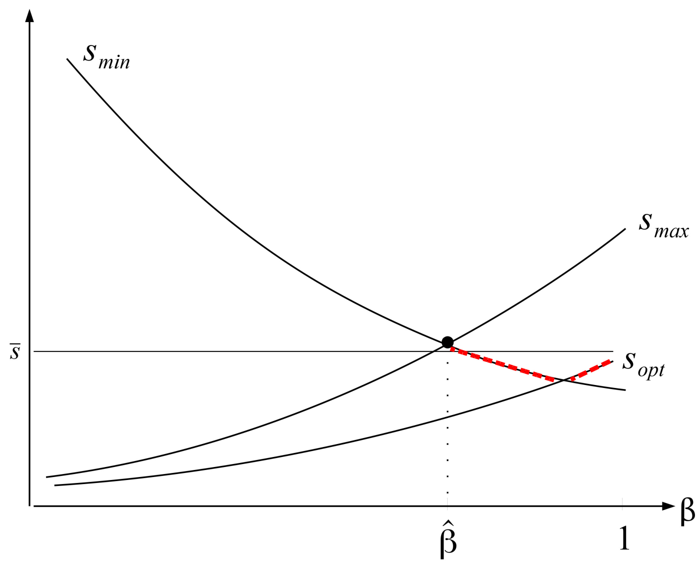

Figure 1: Autarky Equilibrium

Equilibrium savings follow the red dashed line. s_{min} denotes the minimum savings required in period 1 to induce the period 2 self to save. s_{max} denotes the maximum savings that the period 1 self is willing to make, conditional on the nondivisible good being bought. $\bar{s} \equiv \frac{p-1}{2}$ is the path of even savings contributions.

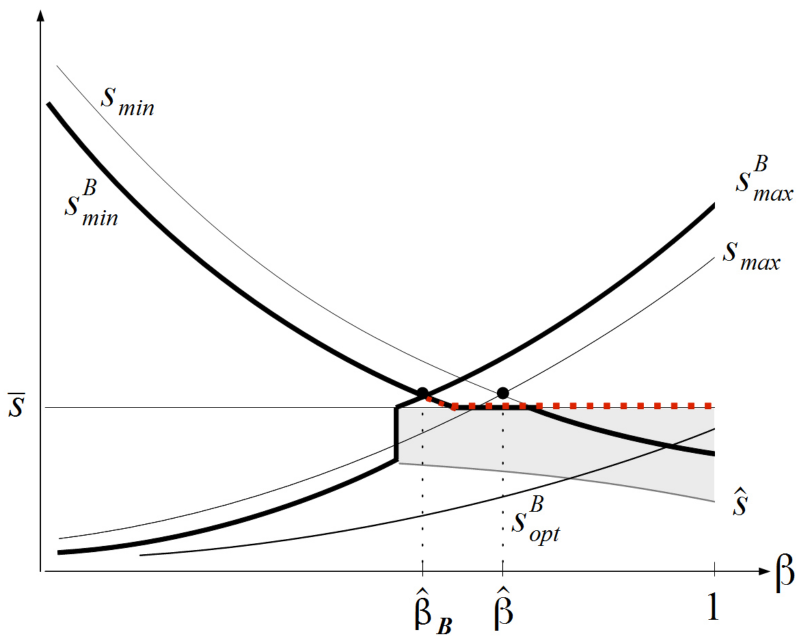

Figure 2: Regular Saver Equilibrium (overcontrol scenario)

Equilibrium savings follow the red dashed line. The nondivisible good can be achieved for $\beta \in [\hat{\beta}_B, 1]$ with the regular saver product, and for $\beta \in [\hat{\beta}, 1]$ in autarky. In an *overcontrol scenario*, the autarky equilibrium savings at the threshold level $\hat{\beta}$ exceed the balanced savings path, $\bar{s} \equiv \frac{p-1}{2}$.

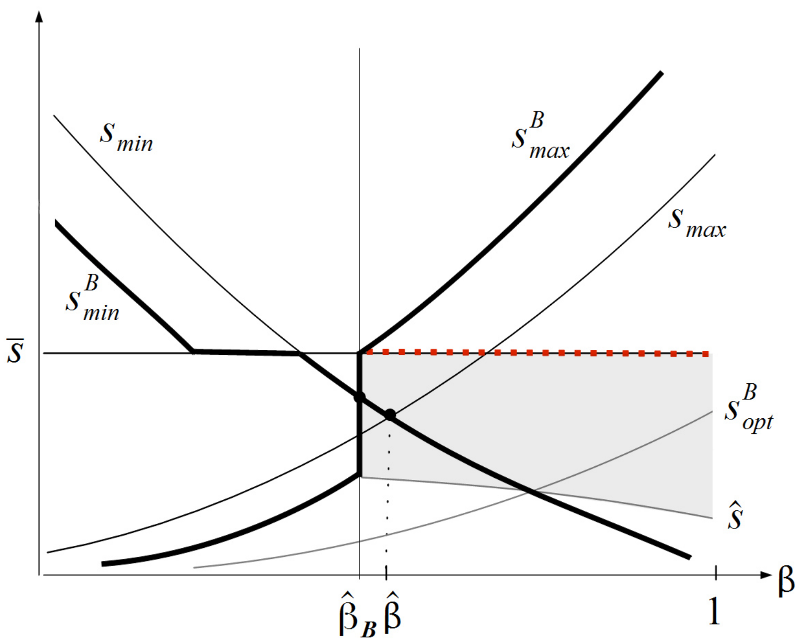

Figure 3: Regular Saver Equilibrium (procrastination scenario)

Equilibrium savings follow the red dashed line. The nondivisible good can be achieved for $\beta \in [\hat{\beta}_B, 1]$ with the regular saver product, and for $\beta \in [\hat{\beta}, 1]$ in autarky. In a *procrastination scenario*, the autarky equilibrium savings at the threshold level $\hat{\beta}$ stay below the balanced savings path, $\bar{s} \equiv \frac{p-1}{2}$.

B Online Appendix: Proofs

Lemma 1. *(a) The period 2 agent is willing to save for the nondivisible and transfer $s_2 = p - 1$ if s_1 is bigger than some threshold value, $s_1 \geq s_{min}$. (b) s_{min} is strictly decreasing in the time-inconsistency parameterβ. (c) The effect of the shock frequency λ on s_{min} is ambiguous.*

Proof. (a) The period 2 agent is willing to save $s_2 = p - 1$ if s_1 is such that

$$u(1 + s_1 - (p-1)) + \beta[(1-\lambda)u(b) + \lambda u(p-1)] \geq u(1 + s_1 - s_2^{No}) + \beta E[u(y_3 + s_2^{No})] \quad (3)$$

It is sufficient to prove that once s_1 is high enough to satisfy the inequality above (i.e., buying the good is optimal), the inequality will also be satisfied for all higher values of s_1. Consider a value s_1' such that buying the good is optimal, then

$$u(2 + s_1' - p) + \beta[(1-\lambda)u(b) + \lambda u(p-1)] \geq u(1 + s_1' - s_2) + \beta E[u(y_2 + s_2)].$$

The inequality holds for all $s_2 < p - 1$, thus it also holds for $s_2^{No}(s_1'')$, the s_2 that is optimal at a higher level $s_1'' > s_1'$, conditional on the nondivisible not being bought. Due to strict concavity of $u(c_t)$,

$$u(1 + s_1' - s_2^{No}(s_1'')) - u(1 + s_1' - (p-1)) \geq u(1 + s_1'' - s_2^{No}(s_1'')) - u(1 + s_1'' - (p-1)),$$

i.e., the consumption gain $(p-1) - s_2^{No}$ from deciding not to save for the good in period 2 gives a higher utility gain when starting from the lower consumption level $1 + s_1'$ than when starting from consumption level $1 + s_1''$. Since

$$\beta[(1-\lambda)u(b) + \lambda u(p-1)] - \beta E[u(y_2 + s_2^{No}(s_1''))] \geq u(1 + s_1' - s_2^{No}(s_1'')) - u(2 + s_1' - p)$$

holds by the optimality of buying the good at s_1', substitution and rearranging yields

$$u(2 + s_1'' - p) + \beta[(1-\lambda)u(b) + \lambda u(p-1)] \quad \geq \quad u(1 + s_1'' - s_2^{No}) + \beta E[u(y_2 + s_2^{No})]$$

for all $s_1'' > s_1'$. Therefore, when s_1 has reached some threshold s_{min}, saving for the nondivisible is optimal for all $s_1 \geq s_{min}$.

(b) For a given β, evaluate inequality 3 at $s_1 = s_{min}$. If β is increased to $\beta' > \beta$, the inequality still holds: $u(b) > u(1 + s_2)$ and $u(p-1) > u(s_2)$ for all $s_2 < p - 1$ given $b > p$. Intuitively, the weight of the reward of saving increases relative to the cost. Since $u'(c) > 0$, the inequality becomes more slack, and will still be satisfied for $s_1' = s_{min} - \epsilon$. Therefore, s_{min} decreases in β.

(c) Investigating the sign of $\partial s_{min}/\partial\lambda$, note that an increase in λ makes it less attractive to save for the nondivisible (which will not be obtained in case of a shock), increasing s_{min}. However, a stronger motive for precautionary savings on the right-hand side decreases the savings difference $(p-1) - s_2^{No}$, which decreases s_{min}. Which effect dominates is a function of $(b - p)$ and $u''(c)$. Formally, both sides of the inequality decrease in λ. As the shock hits, the right-hand side loses 1, at a consumption level $1 + s_2^{No} < p$. The left-hand side loses $(b - p) + 1 > 1$, at a higher consumption level $b > 1 + s_2^{No}$. □

Lemma 2. *The maximum that period 1 would be willing to save, denoted s_{max}, is strictly increasing in the time-inconsistency parameter β.*

Proof. Taking into account that the nondivisible can only be bought if no shock hits in any period, the maximum that period 1 would be willing to pay for its expected purchase (i.e., for $s_2 = p - 1$) can be found by comparing

$$\begin{aligned} u(1-s_1) &+ \beta(1-\lambda)^2(u(2+s_1-p)+u(b)) \\ &+ \beta(1-\lambda)\lambda(u(2+s_1-p)+u(p-1)) \\ &+ \beta\lambda(u(s_1-s_2^{No})+E[u(y_3+s_2^{No})]) \\ &\geq u(1-s_1^{No})+\beta E[u(y_2+s_1^{No}-s_2^{No})+u(y_3+s_2^{No})]. \end{aligned} \tag{4}$$

Define s_{max} as the maximum value of s_1 such that inequality 4 holds (if none exists, let $s_{max} = 0$).

To see the relationship with β, evaluate inequality 4 at $s_1 = s_{max}$. For each side separately, take the derivative w.r.t. β. By the envelope condition, $\frac{dU}{d\beta} = \frac{\partial U}{\partial \beta} + \frac{\partial U}{\partial s_1^{No}} \frac{\partial s_1^{No}}{\partial \beta} = \frac{\partial U}{\partial \beta}$. For a time-inconsistent period 1 agent with $\beta < 1$, only s_1 is a choice variable – s_2^{No} is inferred by backward induction, and depends on β. The resulting derivative of the left-hand side is bigger than the derivative of the right-hand side:

$$\begin{aligned} &(1-\lambda)^2(u(2+s_1-p)+u(b)) \\ &+ (1-\lambda)\lambda(u(2+s_1-p)+u(p-1)) \\ &+ \lambda(u(s_1-s_2^{No})+E[u(y_3+s_2^{No})]) \\ &> E[u(y_2+s_1^{No}-s_2^{No})+u(y_3+s_2^{No})] \end{aligned}$$

Note that $u(1-s_{max}) < u(1-s_1^{No})$ holds by definition of s_{max}. As a result, when s_1 is held constant at s_{max}, and β is increased, the left-hand side increases more than the right-hand side does, so the original inequality is maintained and becomes more slack. The inequality will still hold for $s_1 = s_{max} + \epsilon$. Thus, s_{max} is strictly increasing in β. □

Lemma 3. *The optimal allocation of savings from period 1's perspective, denoted $s_1 = s_{opt}$, is strictly increasing in β, and always smaller than s_{max}.*

Proof. Maximising expected lifetime utility from period 1 perspective, conditional on purchase of the nondivisible (i.e., on $s_2 = p - 1$), yields the following first-order condition for $s_1 = s_{opt}$:

$$\begin{aligned} u'(1-s_{opt}) &= \beta[(1-\lambda)u'(2+s_{opt}-p)+\lambda u'(s_{opt}-s_2^{No})] \\ &+ \beta\lambda\frac{\partial s_2^{No}}{\partial s_1}[-u'(s_{opt}-s_2^{No})+Eu'(y_3+s_2^{No})] \end{aligned}$$

Note that $\partial U_1/\partial s_2^{No} \neq 0$ given $\beta < 1$: Period 1 self does not expect his future self to share his preferences, thus the envelope condition does not apply for s_2^{No}. The first-order condition for s_{opt} can be simplified using the first-order condition from s_2^{No}: $\beta E u'(y_3 + s_2^{No}) = u'(s_1 - s_2^{No})$. Substituting this into the above and simplifying yields

$$u'(1-s_{opt}) = \beta[(1-\lambda)u'(2+s_{opt}-p)+\lambda u'(s_{opt}-s_2^{No})(1+\frac{\partial s_2^{No}}{\partial s_1}\cdot\frac{1-\beta}{\beta})]. \tag{5}$$

Increasing β unambiguously increases the right-hand side of the equation (note $\partial s_2^{No}/\partial s_1 > 0$). To clear, the marginal utility of period 1 consumption must increase, implying an increase in s_{opt}. Thus, s_{opt} increases in β. Further, $s_{opt} \leq s_{max}$, follows by the definition of s_{max}. □

Lemma 4. *For small shock frequencies λ, and in the region where savings are skewed towards period 1, $s_1 \geq \bar{s} \equiv \frac{p-1}{2}$, adopting a regular-instalment product increases the maximum the agent is willing to save, i.e., $s^B_{max} > s_{max}$. A sufficient constraint on the shock frequency is $\lambda < \frac{u'(1)}{u'(0.5)}$. In the region $s_1 < \bar{s}$, adopting the regular-instalment product unambiguously decreases s_{max}.*

Proof. In the region $s_1 \geq \bar{s}$: From inequality 2, the introduction of a penalty D will increase s_{max} whenever $\beta\lambda[u(s_1 - s_2^{No}) - u(s_1 - D - s_2^{No})] < u(1 - s_1^{No}) - u(1 - D - s_1^{No})$. To a first-order approximation, this is equivalent to $\beta\lambda u'(s_1) \cdot D < u'(1) \cdot D$, which holds whenever $\lambda < u'(1)/u'(s_1)$. Given $s_1 \geq \bar{s} > 0.5$, it is sufficient that $\lambda < u'(1)/u'(0.5)$. Therefore, inequality 2 always holds using the original $s_{max}(\beta)$, and it still holds for $s_{max}(\beta) + \epsilon$. For the special case where $D > s_1$, limited liability applies: The left-hand side stays constant as D increases, while the right-hand side decreases in D. The positive effect of D on s_{max} is reinforced. The resulting $s^B_{max}(\beta)$ will be strictly higher than $s_{max}(\beta)$ for $s_1 \geq \bar{s}$.

In the region $s_1 < \bar{s}$: The agent compares

$$\begin{aligned} u(1 - s_1 - D) &+ \beta(1-\lambda)^2(u(2 + s_1 - p) + u(b)) \\ &+ \beta(1-\lambda)\lambda(u(2 + s_1 - p) + u(p-1)) \\ &+ \beta\lambda(u(s_1 - s_2^{No}) + E[u(y_3 + s_2^{No})]) \\ &\geq u(1 - D - s_1^{No}) + \beta E[u(y_2 + s_1^{No} - s_2^{No}) + u(y_3 + s_2^{No})] \end{aligned}$$

With a strictly concave utility function, the utility loss from D when starting at consumption level $1 - s_1$ is bigger than the utility loss from D when starting at consumption level 1: $u(1 - s_1) - u(1 - s_1 - D) > u(1) - u(1 - D)$ for $s_1 > 0$. In other words, the penalty D hurts the agent more when he is saving for the nondivisible than when he is not. With the left-hand side decreasing more than the right-hand side, willingness to save will decrease, shifting the $s^B_{max}(\beta)$-curve below the original $s_{max}(\beta)$-curve for $s_1 < \bar{s}$. Further note s_2^{No} is affected by D, but only through s_1^{No}. For s_1^{No}, the envelope condition applies. □

Lemma 5. *(a) The optimal allocation of savings from period 1's perspective decreases with the adoption of a regular-instalment product, i.e. $s^B_{opt} < s_{opt}$. (b) However, there exists a threshold $\hat{s}(\beta)$ such that saving in the region $\hat{s}(\beta) < s_1 < \bar{s}$ is strictly dominated by saving in balanced instalments, $s_1 = \bar{s}$. (c) As β increases, a larger range of savings $s_1 \in (\hat{s}(\beta), \bar{s})$ is strictly dominated. Equivalently, $\hat{s}(\beta)$ weakly decreases in β.*

Proof. (a) $s^B_{opt} < s_{opt}$ directly follows from equation 5, after allowing for the fact that period 1's consumption is now $c_1 = 1 - s_{opt} - D$.

(b) The threshold $\hat{s}(\beta)$ is the lowest value of s_1 which satisfies

$$\begin{aligned} &u(1 - \hat{s} - D) + \beta(1-\lambda)(u(2 + \hat{s} - p)) + \beta\lambda(u(\hat{s} - s_2^{No}(\hat{s})) + E[u(y_3 + s_2^{No}(\hat{s}))]) \\ &\quad \leq u(1 - \bar{s}) + \beta(1-\lambda)(u(2 + \bar{s} - p)) + \beta\lambda(u(\bar{s} - D - s_2^{No}(\bar{s})) + E[u(y_3 + s_2^{No}(\bar{s}))]). \end{aligned}$$

(c) By construction, $\hat{s}(\beta) < \bar{s} - D$ for all $\beta > 0$. Given $u(1 - \hat{s} - D) > u(1 - \bar{s})$, it must be that

$$\beta(1-\lambda)(u(2+\hat{s}-p)) + \beta\lambda(u(\hat{s} - s_2^{No}(\hat{s})) + E[u(y_3 + s_2^{No}(\hat{s}))])$$
$$<\beta(1-\lambda)(u(2+\bar{s}-p)) + \beta\lambda(u(\bar{s} - D - s_2^{No}(\bar{s})) + E[u(y_3 + s_2^{No}(\bar{s}))]).$$

This inequality will still hold for $\beta' > \beta$, and become more slack. All values of s_1 which were strictly dominated at β are also strictly dominated at β'. The dominated region $(\hat{s}(\beta), \bar{s})$ becomes weakly larger. □

Proposition 1. *Offering a regular saver commitment product makes the nondivisible good achievable under a larger range of preferences, $\beta \in [\hat{\beta}_B, 1]$, instead of $\beta \in [\hat{\beta}, 1]$ in the autarky benchmark. A lower threshold $\hat{\beta}_B \leq \hat{\beta}$ emerges for any positive penalty $D > 0$ if the autarky setting is characterized by pre-emptive overcontrol. In settings characterized by procrastination, a sufficiently large penalty D is required to guarantee $\hat{\beta}_B \leq \hat{\beta}$.*

Proof. In the region $s_1 \geq \bar{s}$, $s^B_{min}(\beta) < s_{min}(\beta)$ and $s^B_{max}(\beta) > s_{max}(\beta)$ (by Lemma 4) trivially imply that $s^B_{max}(\beta) \geq s^B_{min}(\beta)$ holds for all β where $s_{max}(\beta) \geq s_{min}(\beta)$. Using Lemmas 1 and 2, it also holds for $\beta - \epsilon$. Thus, $\hat{\beta}_B \leq \hat{\beta}$ is satisfied for any $D > 0$ whenever starting from an overcontrol scenario (see Figure 2 , characterized by $s_{min}(\hat{\beta}) = s_{max}(\hat{\beta}) > \bar{s}$.

Whenever $s_{min}(\hat{\beta}) = s_{max}(\hat{\beta}) > \bar{s}$ (characterized as a procrastination scenario, see Figure 3), introducing a penalty D shifts s_{max} downward in the region $s_1 < \bar{s}$, implying that $\hat{\beta}_B > \hat{\beta}$ is possible in principle for small penalties D. However, such penalties would decrease welfare in expectation, and would thus not be adopted by fully sophisticated agents. Starting from procrastination scenarios, penalties need to be sufficiently large to raise the maximum that period 1 is willing to save to $\bar{s}$ at the threshold $\hat{\beta}$. In other words, $s^B_{max}(\hat{\beta}) \geq \bar{s}$ guarantees that $\hat{\beta}_B \leq \hat{\beta}$. □

Proposition 2. *For those already able to save in autarky, $\beta \geq \hat{\beta}$, the adoption of a regular saver product weakly smoothes savings contributions (and thus consumption) towards $\bar{s}$.*

Proof. Equilibrium savings in autarky are given by $s_1 = max(s_{min}, s_{opt})$. Starting from procrastination scenarios, commitment works by creating a dominated region $(\hat{s}(\beta), \bar{s})$ that covers the previous savings level s_1, causing the agent to jump to $s_1 = \bar{s}$, and thus achieving balanced savings contributions. Starting from overcontrol scenarios, autarky equilibrium savings are given by $s_1 = s_{min}(\beta)$. From equation 1, any penalty $D > 0$ ensures that $\bar{s} \leq s^B_{min}(\beta) < s_{min}(\beta)$, thus moving savings contributions weakly closer to $\bar{s}$. □

Proposition 3. *For a given shock frequency λ, the minimum effective penalty D_{min} that will enforce saving weakly decreases in the time-inconsistency parameter β.*

Proof. For a given β, $D_{min}(\beta)$ is defined as the minimum penalty such that $s^B_{min}(\beta) \leq s^B_{max}(\beta)$. Holding the penalty D fixed, and increasing β to $\beta' > \beta$, Lemma 1 and 2 assert that $s^B_{min}(\beta') < s^B_{min}(\beta) \leq s^B_{max}(\beta) < s^B_{max}(\beta')$. Thus, penalty $D_{min}(\beta)$ is effective for all $\beta' \geq \beta$. □

Proposition 4. *Optimal Penalty Choice: (a) In autarky settings characterized by procrastination, the minimum penalty required to obtain the nondivisible good, D_{min}, also guarantees perfect consumption smoothing, with $s_1 = \bar{s} \equiv \frac{p-1}{2}$ and $s_2 = 2\bar{s} = p - 1$. Consequently, the optimal contract is to choose D_{min}. (b) In autarky settings characterized by overcontrol, D_{min} fails to guarantee consumption smoothing. The optimal contract involves $D \geq D_{min}$, with equilibrium savings weakly skewed towards period 1 ($s_1 \geq \bar{s}$).*

Proof. Part (a) directly follows from the proof to Proposition 2. For part (b), note that a fully sophisticated agent will never adopt a contract with $D < D_{min}$: This results in $s^B_{max}(\beta) < s^B_{min}(\beta)$, and thus in certain default in period 1, which is dominated by not adopting the product. It trivially follows that when D_{min} results in $s^B_{max}(\beta) \geq s^B_{min}(\beta) > \bar{s}$, the optimal contract involves $D \geq D_{min}$, and achieves equilibrium savings $s_1 \geq \bar{s}$. □

Proposition 5. *Commitment Adoption Decision: (a) Adoption of the Regular Saver product decreases in the shock frequency λ, the nondivisible price p, and the required penalty D_{min}. Adoption increases in the nondivisible good's benefit b. (b) Conditional on being unable to save in autarky ($\beta < \hat{\beta}$), those with the highest amount of time-inconsistency are the least likely to adopt commitment (i.e., adoption increases in β), as the most time-inconsistent agents require the largest penalties.*

Proof. The adoption decision is summarized in the following inequality:

$$\begin{aligned}
&(1-\lambda)^3[u(1-s_1)+u(2+s_1-p)+u(b)] \\
&+(1-\lambda)^2\lambda[u(1-s_1)+(u(2+s_1-p)+u(p-1)] \\
&+(1-\lambda)\lambda[u(1-s_1)+u(s_1-D_{min}-s_2^{No})+E(u(y_3+s_2^{No}))] \\
&+\lambda[u(0)+E(u(y_2-s_2^{No})+u(y_3+s_2^{No}))] \\
&\geq E[u(y_1-s_1^{No})+u(y_2+s_1^{No}-s_2^{No})+u(y_3+s_2^{No})]
\end{aligned} \tag{6}$$

where $s_1 = max\{\bar{s}, s^B_{min}(\hat{\beta}_B)\}$ and $y_t = \{0, 1\}$ depending on the realisation of shocks. The rows of inequality 6 describe the different cases of shock occurrence: The savings plan could be undisturbed by shocks until the end of the agent's life (first row), it could fail in period 3 (second row: period 3 lacks the income to buy the nondivisible), a shock in period 2 could lead to costly default (third row), or a shock in period 1 could prevent saving for the nondivisible altogether (fourth row).

Part (a) of the Proposition follows from the partial derivatives of equation 6. In particular, the shock frequency λ has a first-order effect on the expected benefits from commitment (obtaining benefit *b*), but only a second-order effect on precautionary savings.

Part (b) follows from the fact that the costs of commitment are a function of β (see Proposition 3), while the benefits are not (see footnote 14 for a qualification). □